

1 **Vertical Mixing and the Temperature and Wind Structure of the**
2 **Tropical Tropopause Layer**

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ABSTRACT

5 We show that vertical mixing can lead to significant momentum and heat fluxes in the
6 tropical tropopause layer (TTL) and that these momentum and heat fluxes can force large
7 climatological temperature and zonal wind changes in the TTL. We present the climatology
8 of vertical mixing and associated momentum and heat fluxes as parametrised in the Eu-
9 ropean Centre for Medium Range Weather Forecasting (ECMWF) Interim reanalysis and
10 as parametrised by the mixing scheme currently used in the ECMWF operational analyses.
11 Each scheme produces a very different climatology showing that the momentum and heat
12 fluxes arising from vertical mixing are highly dependent on the scheme used. A dry GCM is
13 then forced with momentum and heat fluxes similar to those seen in ERA-Interim to assess
14 the potential impact of such momentum and heat fluxes. We find a significant response in
15 the TTL, leading to a temperature perturbation of approximately 4 K, and a zonal wind
16 perturbation of approximately 12 m s^{-1} . These temperature and zonal wind perturbations
17 are approximately zonally symmetric, are approximately linear perturbations to the unforced
18 climatology, and are confined to the TTL between approximately 10°N and 10°S . There is
19 also a smaller amplitude tropospheric component to the response. Our results indicate that
20 vertical mixing can have a large but uncertain effect on the TTL, and that choice and im-
21 pact of the vertical mixing scheme should be an important consideration when modelling
22 the TTL.

1. Introduction

Vertical mixing in the free atmosphere has been observed to occur in the tropical tropopause layer (TTL) (Fujiwara et al. 1998; Fujiwara and Takahashi 2001; Fujiwara et al. 2003), with observations indicating substantial exchanges between troposphere and stratosphere due to mixing. The TTL plays an important role in the global climate system (Fueglistaler et al. 2009a), and as such it is fair to ask to what extent vertical mixing could affect the TTL structure.

Fueglistaler et al. (2009b) and Flannaghan and Fueglistaler (2011) find that vertical mixing in the European Centre for Medium Range Weather Forecasting (ECMWF) Interim reanalysis (ERA-Interim) (Simmons et al. 2007; Dee et al. 2011) leads to significant diabatic terms in the TTL. Wright and Fueglistaler (2013) showed that vertical mixing is also important for the diabatic heat budget in other reanalysis datasets, and noted that there are large discrepancies between reanalyses. These studies focused on the heat budget, and therefore temperature tendency, but as noted in Flannaghan and Fueglistaler (2011) there is also a significant momentum forcing due to mixing in ERA-Interim, and so in this study we shall consider the effect of both temperature tendency and momentum forcing.

We shall use two different mixing parametrisation schemes in this study; the scheme used in ERA-Interim and a second scheme that is used in more recent ECMWF models. These two schemes are fundamentally quite different (as discussed in Flannaghan and Fueglistaler (2011)), and give very different results, showing that the forcing terms associated with mixing are highly uncertain. Given the uncertain nature of the forcing that vertical mixing exerts on the atmosphere, it is important to understand the potential impact such terms may have on the atmosphere.

We shall begin by presenting the climatology of the forcing terms generated by each mixing scheme in section 2 and then go on to present the impact of these forcing terms on the TTL climatological temperature and wind in an idealized model in section 3, followed

49 by an analysis of the model results in section 4.

50 2. Climatology of mixing

51 We use ERA-Interim 6 hourly data on a 1° grid on pressure levels chosen to be close to
52 ERA-Interim model levels. For the layer of interest here (the TTL), these pressure levels are
53 very close to, and from the 80 hPa level upwards identical with, the original model levels,
54 such that interpolation errors are minimal.

55 We shall apply two different mixing schemes that are used by ECMWF in the Integrated
56 Forecast System (IFS); the revised Louis (rL) scheme (Louis 1979; Viterbo et al. 1999) and
57 the Monin-Obukhov (MO) scheme as used in current operational analyses (both are defined
58 in Part IV of the IFS documentation.) The definitions of the schemes used in this study are
59 given in full in the Appendix and are also discussed in Flannaghan and Fueglistaler (2011).
60 Both schemes parametrise mixing as a diffusive term, with the diffusivity K referred to here
61 and in the literature as the exchange coefficient. Both schemes allow K to vary as a function
62 of Richardson number Ri defined in terms of the model temperature and wind fields as

$$63 Ri = \frac{N^2}{|\partial \mathbf{u} / \partial z|^2}, \quad (1)$$

64 where N^2 is the static stability and \mathbf{u} is the horizontal wind (u, v) .

65 The MO scheme (or similar variants) is commonly used in global climate models and
66 forecast models, and has the key property that mixing only occurs when the Richardson
67 number Ri falls below approximately 0.25. The rL scheme is used in the IFS models in
68 the lower troposphere, and prior to Cycle 33r1 (introduced in 2008; ERA-Interim is prior
69 to Cycle 33r1) used throughout the free atmosphere, and unlike the MO scheme has a long
70 tail of non-zero K as $Ri \rightarrow \infty$. Flannaghan and Fueglistaler (2011) showed that the long
71 tail of the rL scheme leads to very different mixing than that produced by the MO scheme
72 with a cut-off of mixing at $Ri = 0.25$. We shall not discuss which scheme gives a better
representation of mixing in the TTL, and such a question is non-trivial; the MO scheme *seems*

73 most physical as it features the observed Richardson number cut-off associated with Kelvin-
74 Helmholtz instability, but when gravity waves are not resolved, and therefore not included
75 when computing the Richardson number, using such a cut-off is problematic. Gravity waves
76 are expected to reduce Richardson number, and therefore increase mixing. This mixing is
77 missed when a cut-off at $Ri = 0.25$ is used where Ri is computed without including gravity
78 waves.

79 Both mixing schemes are applied offline to ERA-Interim data. Flannaghan and Fueglistaler
80 (2011) find that applying the ERA-Interim mixing scheme offline matches the diabatic resid-
81 ual output given by ERA-Interim in regions with little convection (where the contribution of
82 mixing is well separated from other diabatic terms in the residual) and so we have confidence
83 in computing mixing offline. See the Appendix for more details and validation of the offline
84 calculation method. We were not able to validate the zonal acceleration forcing exerted on
85 the atmosphere by mixing (due to lack of information on the momentum terms), but we
86 assume that the zonal acceleration forcing can also be computed offline, as the calculations
87 performed are very similar to that for the heating due to the mixing scheme.

88 *a. Zonal mean forcing terms*

89 Figure 1 shows the climatological (1989–2009) annual and seasonal (DJF and JJA) aver-
90 age zonal mean zonal acceleration \bar{X} (where $\bar{\cdot}$ denotes the zonal mean) and the temperature
91 tendency \bar{Q} averaged over the inner tropics (10°N – 10°S), calculated offline from ERA-Interim
92 temperature and wind using the rL and MO schemes.

93 Using the rL scheme, both the zonal mean zonal acceleration and the zonal mean tem-
94 perature tendency have a strong dipole structure in both DJF and JJA, centered at approx-
95 imately 110 hPa. Zonal mean zonal acceleration is largest in JJA, where the dipole has an
96 amplitude of approximately $0.2 \text{ m s}^{-1} \text{ day}^{-1}$ when averaged over the inner tropics. In DJF,
97 dipole structure of the zonal mean zonal acceleration has the opposite sign, and a lower
98 amplitude of approximately $0.1 \text{ m s}^{-1} \text{ day}^{-1}$. As a consequence of the change in sign from

99 DJF to JJA, \overline{X} averaged over the whole period is small everywhere. Zonal mean tempera-
100 ture tendency computed using the rL scheme has a similar structure in both DJF and JJA,
101 with a maximum amplitude of 0.08 K day^{-1} at approximately 90 hPa. \overline{Q} averaged over the
102 whole period has a strong dipole structure. Using the MO scheme, the zonal mean zonal
103 acceleration and zonal mean temperature tendency are both small everywhere, except for
104 the case of zonal mean zonal acceleration in DJF, where we see a dipole structure similar to
105 that when the rL scheme is used.

106 Figure 2 shows the zonal mean latitudinal structure of the forcing terms due to mixing
107 using the rL scheme. Except for zonal acceleration during DJF, the vertical dipole structures
108 shown in Figure 1 are clearly visible in Figure 2, and are confined to the inner tropics (10°N –
109 10°S) with a very symmetric meridional structure about the equator. Note that the dipole
110 produced by the MO scheme in zonal acceleration in DJF also shows a similar latitudinal
111 structure (not shown).

112 Wright and Fueglistaler (2013) show similar dipole structures to those presented in Fig-
113 ure 2 in the average (over all months) zonal mean diabatic heating term in the NCEP, CFSR
114 and JRA reanalyses (see their Figure 6), while MERRA’s diabatic heating from vertical mix-
115 ing is much smaller. The diabatic heating due to mixing in NCEP has a larger magnitude of
116 approximately 0.1 K day^{-1} compared to ERA-Interim (approximately 0.05 K day^{-1} over the
117 inner tropics; see black curve in Figure 1b) and has a broader meridional structure. Both
118 CFSR and JRA have dipole structures confined to the inner tropics with typical magnitudes
119 of approximately 0.03 K day^{-1} in the annual mean value (approximately half the value in
120 ERA-Interim), and with a similar form to that in ERA-Interim.

121 *b. Zonal structure in the forcing terms*

122 As shown by Flannaghan and Fueglistaler (2011), both schemes have very zonally asym-
123 metric distributions of exchange coefficients in the TTL. Here, we shall give the full structure
124 of the exchange coefficient K_{H} and the resulting forcing terms X and Q . We begin with the

125 rL scheme before presenting results using the MO scheme. Figure 3 shows DJF and JJA
 126 averages of K_H computed over 1989 to 2009 using ERA-Interim data. The K_M climatology
 127 is very similar and not shown here (the *Ri* dependence of K_H and K_M is slightly different;
 128 see Appendix.) In DJF, mixing occurs primarily at around 104 hPa with three main regions
 129 of mixing (shown on the figure) over the Maritime Continent (region A), the central Pacific
 130 (region B) and eastern Pacific (region C). In JJA, mixing occurs predominantly over the
 131 Indian Ocean and is co-located with the easterlies associated with the Monsoon circulation.
 132 The mixing in JJA has a deeper vertical structure, with the peak mixing occurring in the
 133 layer centered at 122 hPa.

134 Figure 4 shows the resulting Q and X averaged over the same region and time period as
 135 in Figure 3. In DJF, Q and X are dominated by dipole structures centered at 95 hPa over the
 136 Maritime Continent (region A) and the Eastern Pacific (region C). Q has a peak magnitude
 137 of approximately 0.3 K day^{-1} , and X has a peak magnitude of approximately $1 \text{ m s}^{-1} \text{ day}^{-1}$.
 138 In the zonal mean, there is a high degree of cancellation in X as the dipole structures over
 139 the Maritime Continent (region A) and the Eastern Pacific (region C) have opposite signs,
 140 due to the opposite sign in the background wind shear. Conversely, the dipoles in Q have the
 141 same sign and therefore reinforce each other, explaining the difference in structure between
 142 Figure 2a(i) and (ii). There is no significant temperature tendency or zonal acceleration in
 143 the central Pacific (region B) due to low background shear and low background N^2 here. In
 144 JJA, X and Q are largest over the Indian Ocean region, with a single large dipole structure
 145 centered at 70°E and 113 hPa in both Q and X .

146 Application of the MO scheme to ERA-Interim data gives a very different climatology.
 147 Figure 5 shows the climatology of K_H computed using the MO scheme. When using the
 148 MO scheme, mixing predominantly occurs in the central Pacific (region B) in DJF, with a
 149 maximum exchange coefficient of approximately $10 \text{ m}^2 \text{ s}^{-1}$. This K_H is much higher than
 150 that under the rL scheme (due to difference in nominal mixing lengths between the schemes;
 151 see Appendix), and in this case does result in a small localized zonal acceleration term in this

152 region. Mixing in this region is often very sporadic and is often associated with near zero
153 or negative N^2 . The substantial \overline{X} term in Figure 1a is due to the mixing over the Central
154 Pacific (region B) and also the weaker mixing over the Eastern Pacific (region C). These
155 two regions of mixing have the same sign, and therefore reinforce in the zonal mean, giving
156 rise to a substantial zonal mean despite the local X being smaller in magnitude than those
157 when using the rL scheme. In JJA, we see mixing at 122 hPa over the Maritime Continent
158 (around 120°E). This region has a very background low wind shear, and so the mixing in
159 this region does not result in a large zonal acceleration.

160 **3. Modeling the response to forcing terms**

161 We have shown that substantial forcing terms Q and X can arise from vertical mixing,
162 but that these terms are dependent on and very sensitive to the mixing scheme. Therefore,
163 it is important to understand the order of magnitude of the response to these forcing terms
164 as a measure of the level of uncertainty associated with the representation of vertical mixing
165 in a model. In this section, we shall model the response to idealized forcings with similar
166 structures to the observed climatology of forcing terms arising from the revised Louis scheme
167 shown in Figure 4.

168 *a. Model*

169 We use the Geophysical Fluid Dynamics Laboratory (GFDL) Flexible Modeling System
170 (FMS) spectral dynamical core running at T42 resolution (i.e. approximately 2.8° by 2.8°).
171 Newtonian cooling and Rayleigh damping are applied as specified in Held and Suarez (1994,
172 henceforth referred to as HS94). The equilibrium temperature profile is also that specified
173 in HS94. The Newtonian cooling timescale in the upper troposphere and lower stratosphere
174 is 40 days.

175 We use 60 vertical levels with approximately 800 m resolution in the TTL and lower

176 stratosphere. The vertical levels are distributed

$$\sigma_i = \exp \left[5.5 \left(\frac{i}{n} + \left(\frac{i}{n} \right)^3 \right) \right],$$

177 where σ_i is the i^{th} level in σ -coordinates, (i.e. the pressure on level i is given by $p_i = p_{\text{surf}}\sigma_i$
 178 where p_{surf} is the instantaneous surface pressure), and n is the total number of levels. Here,
 179 $n = 60$. The model top is at 11 scale heights with a sponge layer above 1 hPa.

180 *b. Imposed Diabatic Forcings*

181 We shall impose idealized forcings (both temperature tendency and zonal acceleration)
 182 with similar structures to those observed in section 2, focusing on the dipole structure ob-
 183 served in the zonal mean forcing due to mixing (Figure 1), and on the dipole structure
 184 observed in the Indian Ocean region (shown in Figure 4). We use an idealized zonally
 185 symmetric forcing of the form

$$F^{\text{sym}} = \begin{cases} A \cos\left(\frac{\pi y}{2L_y}\right) \sin\left(\frac{\pi(z_0-z)}{L_z}\right) & \text{where } |y| < L_y, |z| < L_z, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

186 to represent the dipole structure in the zonal mean, where L_y and L_z are the half-widths in
 187 the meridional and vertical directions. z is log-pressure height, and z_0 is the log-pressure
 188 height about which the forcing is located. We choose these parameters such that the forcing
 189 resembles the dipole structure observed in the zonal mean (Figure 1), with $L_y = 10^\circ$ lati-
 190 tude ≈ 1100 km, $L_z = 0.5$ scale heights ≈ 3.5 km and $z_0 = 2.2$ scale heights ≈ 15.5 km \approx
 191 110 hPa. A is the amplitude of the forcing, and will be specified later.

192 We use an idealized forcing of the form

$$F = \begin{cases} \frac{\pi^2 a}{2L_x} \cos\left(\frac{\pi x}{2L_x}\right) F^{\text{sym}} & \text{where } |x| < L_x, \\ 0 & \text{otherwise,} \end{cases} \quad (3)$$

193 to represent the localized dipole structure in the JJA Indian Ocean, where L_x is the half-
 194 width in the zonal direction, and a is the radius of the Earth. We choose $L_x = 30^\circ$ longitude

195 ≈ 3300 km, with the remaining parameters specified as in Eq. (2). The zonal structure of
 196 Eq. (3) is such that $F^{\text{sym}} = \bar{F}$ (where $\bar{\cdot}$ denotes the zonal mean.)

197 We define a local temperature tendency forcing F_Q that has the structure given in Eq. (3)
 198 and a zonal mean amplitude $A = 0.1$ K day $^{-1}$ (chosen to give a similar 10°N–10°S average
 199 zonal mean amplitude of approximately 0.06 K day $^{-1}$ as that in ERA-Interim in JJA shown
 200 in Figure 1). We also define a local zonal acceleration forcing F_X with the same structure
 201 and with $A = 0.3$ m s $^{-1}$ day $^{-1}$ (again, chosen to give a similar amplitude of approximately
 202 0.2 K day $^{-1}$ to that in ERA-Interim in JJA shown in Figure 1). The zonally symmetric
 203 forcings are defined as F_Q^{sym} and F_X^{sym} .

204 We compute an 8000 day control run with no imposed forcing (i.e. with just the HS94
 205 Newtonian cooling and Rayleigh friction). For each forcing, a forced run is then initialized
 206 from the end of the control run, and is again integrated for 8000 days. We define the control
 207 climate to be the average of the last 4000 days of the control run, and the forced climate to
 208 be the average of the last 4000 days of the forced run. We denote the climatological average
 209 over the last 4000 days of each run by $\langle \cdot \rangle$. The last 4000 days of the unforced control run
 210 will be denoted by (T_0, u_0, v_0, w_0) and the last 4000 days of each forced run will be denoted
 211 by (T_1, u_1, v_1, w_1) . The climate perturbation δ to the unforced climate is then defined as

$$\delta x = \langle x_1 \rangle - \langle x_0 \rangle, \quad (4)$$

212 where x is some model variable or derived quantity, such as temperature.

213 *c. Zonal Mean Response to the Imposed Forcings*

214 We shall first present the zonal mean response to the zonally symmetric forcings F_X^{sym} and
 215 F_Q^{sym} , and to the localized forcings F_X and F_Q . Figure 6 shows the zonal mean temperature
 216 response $\delta \bar{T}$ and zonal wind response $\delta \bar{u}$ averaged over 10°N–10°S latitude (referred to here
 217 the inner tropical zonal mean response) to F_X^{sym} , F_Q^{sym} , and both F_X^{sym} and F_Q^{sym} together. We
 218 see that the inner tropical zonal mean temperature response has a dipole structure similar

219 to the dipole forcing structure for all forcings. Figure 6 also shows the inner tropical zonal
 220 mean response to the equivalent localized forcings F_X , F_Q , and both F_X and F_Q together.
 221 We see that the zonal mean inner tropical responses to the localized forcings are very similar
 222 to the equivalent responses to the zonally symmetric forcings, which demonstrates that the
 223 localized solutions are fairly linear. The exception to this similarity is the inner tropical
 224 zonal mean wind response to F_Q and F_Q^{sym} , which show substantial differences above the
 225 100 hPa level, which will be discussed in more detail below.

226 We define the magnitude of the inner tropical zonal mean response as the maximum of
 227 the absolute value of the inner tropical zonal mean response over levels between 130 hPa and
 228 60 hPa. Table 1 summarizes the magnitudes of the responses shown in Figure 6. Again, we
 229 note that the zonally symmetric forcing gives a very similar magnitude response to the zonally
 230 localized forcings. We also see that the temperature response to both forcings is similar to
 231 the sum of the responses to each forcing. Again, this indicates that the responses are fairly
 232 linear. Both the temperature and wind responses are dominated by the response to F_X ,
 233 which is responsible for approximately 65% of the temperature response to both forcings
 234 and for almost all of the zonal wind response. The combined forcings yield a response
 235 of approximately 3.5 K, which is highly significant in the context of tropical tropopause
 236 temperatures and stratospheric water vapor.

237 Figure 7 shows $\delta\bar{T}$ and $\delta\bar{u}$ for all of the forcings above. As above, the responses to the
 238 symmetric forcing and the equivalent localized forcing are very similar, and the response
 239 to F_X is larger than the response to F_Q , with the response to both forcings dominated by
 240 the response to F_X . We see that the responses to F_X and F_X^{sym} are largest in the inner
 241 tropics (10°N–10°S where the forcing is largest) but we see a wider response in the lower
 242 stratosphere, with the cold anomaly at 70 hPa extending to approximately 20° latitude. The
 243 dipole structure in the response is at a slightly higher altitude than the forcing, and the cold
 244 anomaly extends above the forced region. There is also a response in the upper troposphere
 245 that is strongest at 30° latitude. This tropospheric response is very similar to that shown in

246 Garfinkel and Hartmann (2011).

247 The responses to F_Q and F_Q^{sym} shown in Figure 7 (panels b and e) have a wider latitudinal
248 structure than the responses to F_X and F_X^{sym} , extending to approximately 15° . The zonal
249 wind responses to F_Q and F_Q^{sym} are quite different, with an order 4 m s^{-1} zonal wind response
250 to F_Q , but little response to F_Q^{sym} .

251 *d. Zonally Asymmetric Response to Localized Forcings*

252 Figure 8 shows the zonally asymmetric response to local forcings F_X , F_Q , and both
253 F_X and F_Q in the inner tropics (10°N – 10°S). We see that both responses are quite zonally
254 symmetric, and as such we do not emphasize the zonally asymmetric structure of the response
255 to localized forcings in this paper, and will only describe the structure briefly.

256 The response to F_X is particularly zonally symmetric, with strong winds of up to 12 m s^{-1}
257 at around 100 hPa. Winds are strongest in the forced region. The thermal wind temperature
258 response has more asymmetry (due to changing latitudinal structure, not shown here). The
259 response to F_Q is less symmetric, and resembles a stationary Kelvin wave. Given appropriate
260 easterly zonal winds in the TTL, the imposed forcing can excite a stationary Kelvin wave
261 (one that propagates at the same speed as the background wind, and so is stationary when
262 Doppler shifted) if the vertical structure of the forcing is close to the stationary Kelvin wave
263 vertical structure. This stationary wave propagates vertically from the forced region into the
264 stratosphere, and decelerates the stratosphere at around 50 hPa in Figure 8b. The stationary
265 wave accelerates the forced region, and is therefore also responsible for the westerly wind
266 response to F_X from 100 hPa to 50 hPa in Figures 6 and 7e. The response to both F_X and
267 F_Q shown in Figure 8c is close to the linear superposition of the two solutions. Most of the
268 zonal asymmetry comes from the response to F_Q , leading to the strongest wind responses
269 away from the forced region.

270 4. Interpretation of Results

271 In the following, we will focus on the zonally symmetric cases F_Q^{sym} and F_X^{sym} since
 272 the asymmetric forcings give similar responses in the zonal mean (see section 3) and give
 273 very similar results in the analysis presented below (not shown). In order to investigate
 274 the responses to the zonally symmetric forcings, we analyze the time mean zonal mean
 275 momentum and buoyancy equations.

276 a. Response to Imposed Heating

277 The imposed heating F_Q^{sym} forces the time mean zonal mean buoyancy equation (Andrews
 278 et al. 1987, p124),

$$\frac{\partial \bar{\theta}}{\partial t} + a^{-1} \bar{v} \frac{\partial \bar{\theta}}{\partial \phi} + \bar{w} \frac{\partial \bar{\theta}}{\partial z} = -\tau^{-1} (\bar{\theta} - \theta_{eq}) + e^{\kappa z/H} F_Q^{\text{sym}} - \frac{1}{a \cos \phi} \frac{\partial (\bar{\theta}' v' \cos \phi)}{\partial \phi} - \frac{1}{\rho_0} \frac{\partial (\rho_0 \bar{\theta}' w')}{\partial z}, \quad (5)$$

279 where $\kappa = R/c_p$, ϕ is latitude, τ is the Newtonian cooling timescale (40 days in this study),
 280 θ is the potential temperature, ρ_0 is the log-pressure density and a is the Earth's radius. The
 281 $\partial \bar{\theta} / \partial t$ term disappears when we take the climatological mean. The climatological means of
 282 the remaining terms (computed offline) averaged over $\pm 10^\circ$ latitude are shown in Figure 9a.
 283 The budget is not perfectly closed due to the offline nature of the calculation, but the errors
 284 are small. We see that the vertical advection term and the Newtonian cooling term are the
 285 dominant balance.

286 Figure 9b shows the difference between the unforced and forced runs in these terms. We
 287 see that

$$\delta \left(\bar{w} \frac{\partial \bar{\theta}}{\partial z} \right) + \tau^{-1} \delta \bar{\theta} \approx e^{\kappa z/H} F_Q^{\text{sym}}, \quad (6)$$

288 with both of these terms of a similar order of magnitude. All of the remaining terms do not
 289 significantly change from the unforced run to the forced run and so do not contribute to the
 290 response.

291 As noted in section 3, the responses to F_X^{sym} and F_Q^{sym} are approximately linear. There-
 292 fore, we write the change in vertical advection in terms of the base climatology and the

293 change in the climatology, as

$$\delta \left(\bar{w} \frac{\partial \bar{\theta}}{\partial z} \right) = \langle \bar{w}_0 \rangle \frac{\partial \delta \bar{\theta}}{\partial z} + \delta \bar{w} \frac{\partial \langle \bar{\theta}_0 \rangle}{\partial z} + \delta \bar{w} \frac{\partial \delta \bar{\theta}}{\partial z}.$$

294 The first of these linear terms is dominant (except below 140 hPa, where the second term is
295 of a similar order of magnitude to the first term), so

$$\left(\langle \bar{w}_0 \rangle \frac{\partial}{\partial z} + \tau^{-1} \right) \delta \bar{\theta} \approx e^{\kappa z/H} F_Q^{\text{sym}}. \quad (7)$$

296 Therefore, as a parcel rises due to the climatological upwelling $\langle \bar{w}_0 \rangle$ it is warmed by the
297 positive region of F_Q^{sym} , cools radiatively, then is further cooled by the negative region of
298 F_Q^{sym} before returning to the unforced solution above the forcing region by radiative heating.
299 This explains the phase lag in the vertical between the forcing structure and the temperature
300 response that can be seen in Figure 9b. Eq. (7) shows that either increasing the climatolog-
301 ical upwelling $\langle \bar{w}_0 \rangle$ or reducing the Newtonian cooling timescale τ would lead to a reduction
302 in the temperature amplitude of the response to F_Q^{sym} .

303 The zonal wind response to F_Q^{sym} is in thermal wind balance with the temperature re-
304 sponse, so using the thermal wind equation near the equator (given by Andrews et al. 1987,
305 p318),

$$\frac{\partial \delta \bar{u}}{\partial z} \approx - \frac{R}{H \beta a^2} \frac{\partial^2 \delta \bar{T}}{\partial \phi^2}. \quad (8)$$

306 *b. Response to Imposed Zonal Acceleration*

307 We use a similar analysis here as was used above for the response to the imposed heating,
308 but analyzing the zonal mean zonal momentum equation (Andrews et al. 1987, p124),

$$\frac{\partial \bar{u}}{\partial t} + \bar{v} \left[\frac{1}{a \cos \phi} \frac{\partial (\bar{u} \cos \phi)}{\partial \phi} - f \right] + \bar{w} \frac{\partial \bar{u}}{\partial z} = F_X^{\text{sym}} - \frac{1}{a \cos^2 \phi} \frac{\partial (\overline{u'v'} \cos^2 \phi)}{\partial \phi} - \frac{1}{\rho_0} \frac{\partial (\rho_0 \overline{u'w'})}{\partial z}. \quad (9)$$

309 Again, the $\partial \bar{u} / \partial t$ term disappears when we compute the climatological mean of this equation,
310 and we show the climatological means of the remaining terms in Figure 10a. The zonal
311 momentum budget is less straightforward than the heat budget above as all the terms have

312 similar orders of magnitude. However, when we compute the difference between the forced
 313 and unforced runs (Figure 10b) we see that the only term to significantly change is the
 314 vertical advection term, so

$$\delta \left(\overline{w} \frac{\partial \overline{u}}{\partial z} \right) \approx F_X^{\text{sym}}.$$

315 Shaw and Boos (2012) force a dry GCM with a localized zonal acceleration forcing in the
 316 upper troposphere and also find that the vertical advection term is important (they discuss
 317 the equivalent term in the vorticity equation).

318 As above, we can write the change in vertical advection in terms of the base climatology
 319 and the change in the climatology, as

$$\delta \left(\overline{w} \frac{\partial \overline{u}}{\partial z} \right) = \langle \overline{w_0} \rangle \frac{\partial \delta \overline{u}}{\partial z} + \delta \overline{w} \frac{\partial \langle \overline{u_0} \rangle}{\partial z} + \delta \overline{w} \frac{\partial \delta \overline{u}}{\partial z}.$$

320 The first of these linear terms is dominant, so

$$\langle \overline{w_0} \rangle \frac{\partial \delta \overline{u}}{\partial z} \approx F_X^{\text{sym}}. \quad (10)$$

321 The zonal wind response to F_X^{sym} can therefore be explained by considering a parcel of air
 322 rising due to the climatological upwelling $\langle \overline{w_0} \rangle$ that is first accelerated by the positive region
 323 of F_X^{sym} and is then decelerated by the negative region of F_X^{sym} . This explains the single-
 324 signed form of the zonal wind response to the dipole forcing. From Eq. (10), we see that an
 325 increase in climatological vertical wind $\langle \overline{w_0} \rangle$ would reduce the amplitude of the response $\delta \overline{u}$
 326 to F_X^{sym} , with $\delta \overline{u} \sim \langle \overline{w_0} \rangle^{-1}$.

327 5. Conclusions

328 We have calculated the diabatic heating and zonal acceleration due to mixing based on
 329 two parametrisations of shear-flow mixing. We find a substantial heating and acceleration
 330 in the TTL. These forcing terms take a dipole structure confined to the inner tropics, and
 331 are strongest in boreal summer over the Indian Ocean. The climatological heating and ac-
 332 celeration terms in ERA-Interim are largest in boreal summer over the Indian Ocean, with

333 amplitudes of 0.5 K day^{-1} and $2 \text{ m s}^{-1} \text{ day}^{-1}$ respectively. In the zonal mean averaged over
334 the inner tropics, the magnitude of the heating and acceleration terms are 0.08 K day^{-1} and
335 $0.2 \text{ m s}^{-1} \text{ day}^{-1}$. We have used a dry dynamical core to calculate the response to forcings
336 similar to those found in the climatology of ERA-Interim, and find remarkably large re-
337 sponses in temperature and zonal wind. Forcings of a similar magnitude to those found in
338 ERA-Interim during JJA produce a 4 K temperature response and a 12 m s^{-1} zonal wind
339 response in the TTL. Such a temperature response would have a large effect on water va-
340 por entering the stratosphere, changing TTL water vapor concentration by approximately
341 2 ppmv (roughly 75% of the current mixing ratio for air entering the lower stratosphere;
342 Fueglistaler and Haynes 2005).

343 Further, we find that the amplitude of the response is dependent on the mean upwelling
344 $\langle \bar{w}_0 \rangle$, and that the amplitude of the response to heating (a comparatively small proportion
345 of the response to both heating and forcing; see section 2) is also dependent on the radiative
346 timescale τ . We therefore compare $\langle \bar{w}_0 \rangle$ and τ between the background climatology of the
347 dry GCM and ERA-Interim to assess whether the response is likely to be similar for a realistic
348 base state.

349 Figure 11 shows the climatology of \bar{w} in ERA-Interim, along with the upwelling $\langle \bar{w}_0 \rangle$
350 from the background model run. We see that below the 100 hPa level, the model upwelling
351 is approximately 2 to 3 times smaller than the annual mean upwelling in ERA-Interim,
352 but above the 100 hPa level, model upwelling is similar to the annual mean upwelling in
353 ERA-Interim. Dee et al. (2011) note that the mean vertical transport velocity in ERA-
354 Interim is greater than water vapor observations suggest (Schoeberl et al. 2008) in the lower
355 stratosphere, so model upwelling $\langle \bar{w}_0 \rangle$ may be larger than in reality above the 100 hPa level.
356 We can therefore conclude that the response to the forcing with a more realistic basic state is
357 likely to be smaller below the 100 hPa level, but similar or possibly larger above the 100 hPa
358 level.

359 Upwelling in ERA-Interim has a clear annual cycle, with a minimum in upwelling at

360 100 hPa in September, and a maximum upwelling at 100 hPa in boreal winter, in agreement
361 with Randel et al. (2007). The minimum upwelling in ERA-Interim in JJA coincides with
362 the largest forcing from the mixing scheme (both locally over the Indian Ocean and also in
363 the zonal mean; see Figures 1 and 2), potentially amplifying the response to mixing in the
364 summer, and suppressing the response to mixing in the winter, leading to a large annual
365 cycle in the response to mixing. This relationship would be interesting to investigate in a
366 future study.

367 The τ used in the model here (as specified in HS94) is 40 days. We have used the Fu and
368 Liou (1992) radiation scheme with perturbations of a similar vertical scale to the responses
369 shown in section 3, and find that τ varies with height, is approximately 15 days at 100 hPa,
370 and decreases with height into the stratosphere (not shown). This indicates that the τ used
371 in the model in this study is too long, and that the true response to the forcing should
372 be smaller. In section 4 we showed that τ only affects the amplitude of the response to
373 the imposed heating, and this is the smaller component of the response to both forcings.
374 Therefore, we expect that changing τ would have only a small affect on the overall response.

375 Taking the corrections mentioned above into account, we would expect that the response
376 to vertical mixing in ERA-Interim and similar models to be order 2 K to 4 K, and order
377 6 m s^{-1} to 12 m s^{-1} in the boreal summer. This is a substantial response in the context of
378 TTL temperatures and winds.

379 The modeling study presented here uses a steady state forcing that has a similar average
380 structure to the forcing in ERA-Interim during JJA. In reality, the forcing strongly varies
381 with time and is very intermittent (see Flannaghan and Fueglistaler (2011)). However,
382 the model's response to the forcing is quite linear. Consequently, we do not expect that
383 this simplification substantially alters the nature of the solution. Similarly, we have not
384 investigated the solution to a slowly varying annual cycle in forcing. The timescales of the
385 solution are the advection timescale and the Newtonian cooling timescale. The timescale for
386 vertical advection in reality is of order 2 to 3 months (Fueglistaler et al. 2009a). As noted

387 above, there is an annual cycle in \bar{w} , and therefore there is also some seasonal variation
388 in the advection timescale. The timescale of Newtonian cooling τ is set as $\tau = 40$ days,
389 whereas in reality a reasonable estimate is $\tau \approx 15$ days. Clearly the Newtonian cooling
390 timescale is shorter than the interseasonal variability in the forcing terms and so would not
391 be expected to be important for interseasonal variability. The advection timescale however
392 is sufficiently long to suggest that interseasonal variability would significantly affect the
393 solution. To investigate the effect of interseasonal variability further, a model with more
394 reasonable upwelling velocities (and annual cycle in upwelling) would be needed, and so is
395 beyond the scope of this study. However, investigating the effect of interseasonal variability
396 of the background state is an important study to perform as it could significantly alter the
397 magnitude and seasonality of the response.

398 Mixing schemes are a modeling detail that are not often discussed with respect to studies
399 of the TTL, and are sometimes used as tuning parameters. We have shown that these mixing
400 schemes have the potential to produce significant impacts on the climate of the model,
401 highlighting the particular importance of mixing schemes to TTL winds and temperatures
402 in climate models. Mixing has been observed to occur in the TTL and can be very intense
403 (Fujiwara et al. 1998; Fujiwara and Takahashi 2001; Fujiwara et al. 2003), and so it is possible
404 that mixing could have a significant effect the climate of the TTL in reality.

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407 Dynamics Laboratory for providing the model used in this study and for providing the
408 computer time to perform the model runs. We thank ECMWF for providing the ERA-
409 Interim data.

APPENDIX

410

411 **Mixing scheme definitions**

412 Parametrisation schemes typically approximate mixing as a diffusive process, with the
 413 diabatic tendency due to mixing given by

$$\rho \left(\frac{\partial \phi}{\partial t} \right)_{\text{mix}} = \frac{\partial}{\partial z} \left(\rho K_\phi \frac{\partial \phi}{\partial z} \right), \quad (\text{A1})$$

414 where ϕ is the quantity being mixed (dry static energy when computing heat fluxes and tem-
 415 perature tendency or horizontal wind when computing momentum fluxes and acceleration),
 416 and K_ϕ is the exchange coefficient.

417 The parametrisation defines K_ϕ in terms of the bulk (grid-scale) quantities, and here is
 418 defined as

$$K_\phi = \ell^2 \left| \frac{\partial \mathbf{u}}{\partial z} \right| f_\phi(Ri). \quad (\text{A2})$$

419 Here, ℓ is the nominal mixing length, and dimensionalizes the equation.

420 *a. Monin-Obukhov-motivated (MO) scheme*

421 The ECMWF IFS has, since Cycle 33 (IFS Cy33r1), used a scheme that is inspired
 422 by the solution given by Monin and Obukhov (1954) to the problem of boundary layer
 423 turbulence, but is applied throughout the free atmosphere (Nieuwstadt 1984). This scheme
 424 is qualitatively similar to the scheme used in the NCAR CAM4 model (Bretherton and Park
 425 2009).

426 In statically stable conditions, where $Ri > 0$, the exchange coefficients K_M and K_H for
 427 momentum and heat are defined by Eq. (A2) with

$$f_M(Ri) = (1 + 5\zeta)^{-2}, \quad (\text{A3a})$$

$$f_H(Ri) = \frac{1}{(1 + 5\zeta)(1 + 4\zeta)^2}, \quad (\text{A3b})$$

428 where ζ is a non-dimensional function of Ri , defined as the solution to

$$429 \quad Ri = \frac{\zeta(0.74 + 4.7\zeta)}{(1 + 4.7\zeta)^2}, \quad (\text{A4})$$

430 which is a fit to observational data given in Businger et al. (1971). When $Ri < 0$ (statically unstable conditions),

$$431 \quad f_M(Ri) = (1 - 16Ri)^{1/2}, \quad (\text{A5a})$$

$$432 \quad f_H(Ri) = (1 - 16Ri)^{3/4}. \quad (\text{A5b})$$

433 The nominal mixing length, ℓ , is set at a constant value of 150 m in the MO scheme. Figure 12 shows f_M and f_H as a function of Richardson number Ri as defined in this section.

434 Eq. (A3) and Eq. (A5) are taken from the ECMWF IFS Cy33r1 documentation, and Eq. (A3) is very similar to the equivalent relation given by Businger et al. (1971), although 435 not exactly the same. In the ECMWF IFS Cy33r1 documentation, the definition of ζ is not given, and so the definition of ζ given by Eq. (A4) is taken from Businger et al. (1971). We 436 expect the equivalent relation in the IFS parametrisation to be similar.

437 *b. Revised Louis (rL) scheme*

438 The ECMWF IFS model prior to Cycle 33, including the version used in the ECMWF ERA-Interim project (IFS Cy31r2) (Dee et al. 2011), uses a different scheme, which was 439 originally devised to be numerically simple to compute, but is used in IFS Cy31r2 because it increases the amount of mixing in the lower troposphere, which was absent when using the 440 MO scheme. The scheme used is a revised version of the Louis scheme (Louis 1979), and is 441 given as

$$442 \quad f_M(Ri) = \frac{1}{1 + 10Ri(1 + Ri)^{-1/2}}, \quad (\text{A6a})$$

$$443 \quad f_H(Ri) = \frac{1}{1 + 10Ri(1 + Ri)^{1/2}}, \quad (\text{A6b})$$

444 when $Ri > 0$. When $Ri < 0$, f_M and f_H are the same as given above for the MO scheme in Eq. (A5).

447 The nominal mixing length ℓ is approximately 40 m in the rL scheme. Here, ℓ depends
448 on height, but over the TTL it is approximately constant, and for this study it is sufficient
449 to use a value of 40 m.

450 Figure 12 shows f_M and f_H for both the MO and rL schemes. We see that the rL scheme
451 has a long tail, with significant mixing occurring even at $Ri \sim 1$. The long tail of the
452 rL scheme contributes a lot of additional mixing compared with the MO scheme. However,
453 $\ell \approx 40$ m in the TTL in the rL scheme but $\ell = 150$ m in the MO scheme, resulting in
454 similar average exchange coefficients for both schemes. Other mixing schemes, such as the
455 scheme used in NCAR CAM3, are qualitatively similar to the rL scheme, with no cut-off in
456 Richardson number (Bretherton and Park 2009).

457 Validation of Offline Scheme

458 ERA-Interim provides a total diabatic heating output, and a total radiative heating
459 output (including the radiative contribution from clouds). The difference of these two fields,
460 the residual diabatic temperature tendency, gives the contribution from all non-radiative
461 diabatic processes, which are predominantly latent heating and mixing, shown by Fueglistaler
462 et al. (2009b). Unfortunately these are not available separately. To test the validity of
463 applying the mixing scheme offline, we compare the residual diabatic temperature tendency
464 in ERA-Interim with the temperature tendency predicted by the offline mixing scheme.

465 Figure 13 shows the zonal mean ECMWF residual diabatic temperature tendency, the
466 temperature tendency predicted by the offline mixing scheme, and the difference between
467 these two quantities averaged over 1 January 2000 to 20 January 2000 averaged over 10°N
468 to 10°S . In all results presented here, the mixing scheme is applied to the data before any
469 averaging takes place. This is essential as the mixing schemes are highly non-linear. We see
470 that below the 100 hPa level, there is a large positive temperature tendency in the ERA-
471 Interim residual that is not captured by the mixing scheme. This is due to convection and

472 the associated latent heat release. Above the 100 hPa level, the residual is slightly more
473 negative than that predicted by the mixing scheme; this is due to convective cold tops. In
474 regions of no convection, the offline mixing calculation fits the residual term very well, with
475 errors of approximately 10% throughout the TTL (Flannaghan and Fueglistaler 2011), and
476 so we conclude that the offline application of the mixing scheme can be expected to give a
477 fair representation of the model vertical mixing throughout the TTL.

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572 **List of Tables**

- 573 1 The magnitude of the inner tropical zonal mean response to zonally symmetric
574 forcing and zonally localized forcing. The magnitude of the inner tropical
575 zonal mean response is defined as the maximum of the absolute value of the
576 inner tropical zonal mean response over levels between 130 hPa and 60 hPa. 27

Forcing type	Symmetric forcing		Localized forcing	
	Temp, K	Wind, m s ⁻¹	Temp, K	Wind, m s ⁻¹
F_X	2.8	11.9	2.5	12.1
F_Q	1.4	1.3	1.3	3.2
F_X and F_Q	3.3	11.9	3.7	12.7

TABLE 1. The magnitude of the inner tropical zonal mean response to zonally symmetric forcing and zonally localized forcing. The magnitude of the inner tropical zonal mean response is defined as the maximum of the absolute value of the inner tropical zonal mean response over levels between 130 hPa and 60 hPa.

577 List of Figures

- 578 1 Climatological mean profiles (1989–2009) averaged over 10°N–10°S of a) Zonal
579 mean zonal acceleration, \overline{X} , and b) zonal mean temperature tendency, \overline{Q} , for
580 DJF (green), JJA (blue) and the annual average (black). Diabatic terms are
581 computed using the rL scheme (solid) and the MO scheme (dashed). 31
- 582 2 Climatological zonal mean a) zonal acceleration and b) temperature tendency
583 for i) DJF and ii) JJA computed using the rL scheme applied to ERA-Interim
584 data from 1989 to 2009. 32
- 585 3 Climatologically averaged exchange coefficient K_H according to the rL scheme
586 for a) DJF and b) JJA averaged over 10°S–10°N using ERA-Interim data from
587 1989 to 2009. Black contours show zonal wind. The labeled regions (“A”, “B”,
588 “C”) of mixing in panel a) are referred to in the text. 33
- 589 4 a) Temperature tendency Q and b) zonal acceleration X due to the forcing
590 terms arising from the revised Louis mixing scheme for i) DJF and ii) JJA
591 averaged over 10°S–10°N using ERA-Interim data from 1989 to 2009. Black
592 contours show zonal wind. The regions A, B and C shown here are the same
593 as those in Figure 3. 34
- 594 5 As in Figure 3 but using the MO scheme. Region B is marked in the same
595 location as in Figure 3a. Note that the color scale has been chosen to saturate
596 before the maximum K_H in the DJF Pacific (approximately $10 \text{ m}^2 \text{ s}^{-1}$; regions
597 above $3 \text{ m}^2 \text{ s}^{-1}$ are shown in white) to highlight the structure of K_H elsewhere
598 in the domain. 35
- 599 6 Zonal average response to zonally symmetric forcings F_X^{sym} (blue solid), F_Q^{sym}
600 (red solid) and both F_X^{sym} and F_Q^{sym} (black solid), in a) temperature and b)
601 zonal wind, averaged over 10°N–10°S, with HS94 background state. Similarly,
602 the responses to the local forcings F_X (blue dashed), F_Q (red dashed) and both
603 F_X and F_Q (black dashed). 36

- 604 7 Zonal mean temperature response $\delta\bar{T}$ (colors) and zonal mean wind response
605 $\delta\bar{u}$ (black contours; contour spacing 2 m s^{-1} , 1 m s^{-1} in b and e) for forcings
606 a) F_X^{sym} , b) F_Q^{sym} , c) both F_X^{sym} and F_Q^{sym} , d) F_X , e) F_Q , and f) both F_X and
607 F_Q , with HS94 background state. The temperature color scale used in b) and
608 e) is half that shown in the color bar. White contours show the structure of
609 the forcing (normalized by amplitude.) 37
- 610 8 Inner tropical (10°N – 10°S) average temperature response δT (colors) and
611 zonal wind response δu (black contours; contour spacing 2 m s^{-1} , 1 m s^{-1}
612 in b) for localized forcings a) F_X , b) F_Q , and c) both F_X and F_Q , with HS94
613 background state. The temperature color scale used in b) is half that shown
614 in the color bar. White contours show the structure of F_X and F_Q . 38
- 615 9 a) Profiles of the terms in Eq. (5) (the time mean zonal mean buoyancy
616 equation) averaged over $\pm 10^\circ$ for the run forced with F_Q^{sym} ; the meridional
617 advection term (blue solid), the vertical advection term (green solid), the
618 meridional eddy heat flux term (blue dashed), the vertical eddy heat flux term
619 (green dashed), the Newtonian cooling term (red) and the imposed heating
620 F_Q^{sym} (black solid) are shown. The sign of all terms except F_Q^{sym} are chosen to
621 put them on the LHS of Eq. (5). The black dotted line shows the sum of all
622 terms that balance the forcing term. b) Profiles of the difference between the
623 forced and unforced runs for each quantity shown in a). 39

- 624 10 a) Profiles of the terms in Eq. (9) (the time mean zonal mean zonal momentum
625 equation) averaged over $\pm 10^\circ$ for the run forced with F_X^{sym} ; the sum of the
626 meridional advection and Coriolis terms (blue solid), the vertical advection
627 term (green solid), the meridional eddy momentum flux term (blue dashed),
628 the vertical eddy momentum flux term (green dashed) and the imposed zonal
629 acceleration F_X^{sym} (black solid) are shown. The black dotted line shows the
630 sum of all terms that balance the forcing term. The sign of all terms except
631 F_X^{sym} are chosen to put them on the LHS of Eq. (9). b) Profiles of the
632 difference between the forced and unforced runs for each quantity shown in a). 40
- 633 11 Mean vertical velocity \bar{w} averaged over 10°N to 10°S (solid lines) and at the
634 equator (dashed lines) for the HS94 run with no imposed forcing (black), and
635 for ERA-Interim averaged over 1979 to 2012 (blue lines, thick line is average,
636 thin lines are climatological annual cycle monthly averages.) 41
- 637 12 $f_M(Ri)$ (black) and $f_H(Ri)$ (blue) in the MO scheme (solid) and rL scheme
638 (dashed). 42
- 639 13 Zonal mean ERA-Interim residual diabatic temperature tendency (solid), the
640 temperature tendency due to vertical mixing as parametrised by the rL scheme
641 (dashed), and the difference between these curves (dash-dot) averaged over
642 January 2001 over 10°S – 10°N . 43

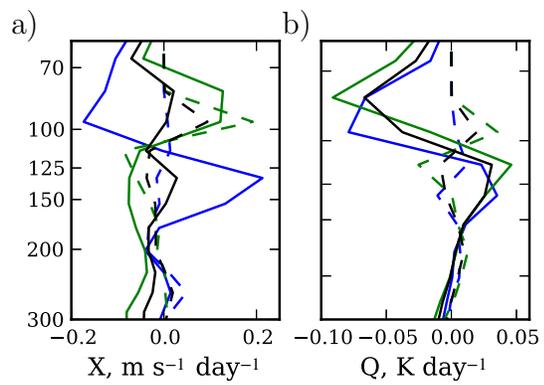


FIG. 1. Climatological mean profiles (1989–2009) averaged over 10°N–10°S of a) Zonal mean zonal acceleration, \bar{X} , and b) zonal mean temperature tendency, \bar{Q} , for DJF (green), JJA (blue) and the annual average (black). Diabatic terms are computed using the rL scheme (solid) and the MO scheme (dashed).

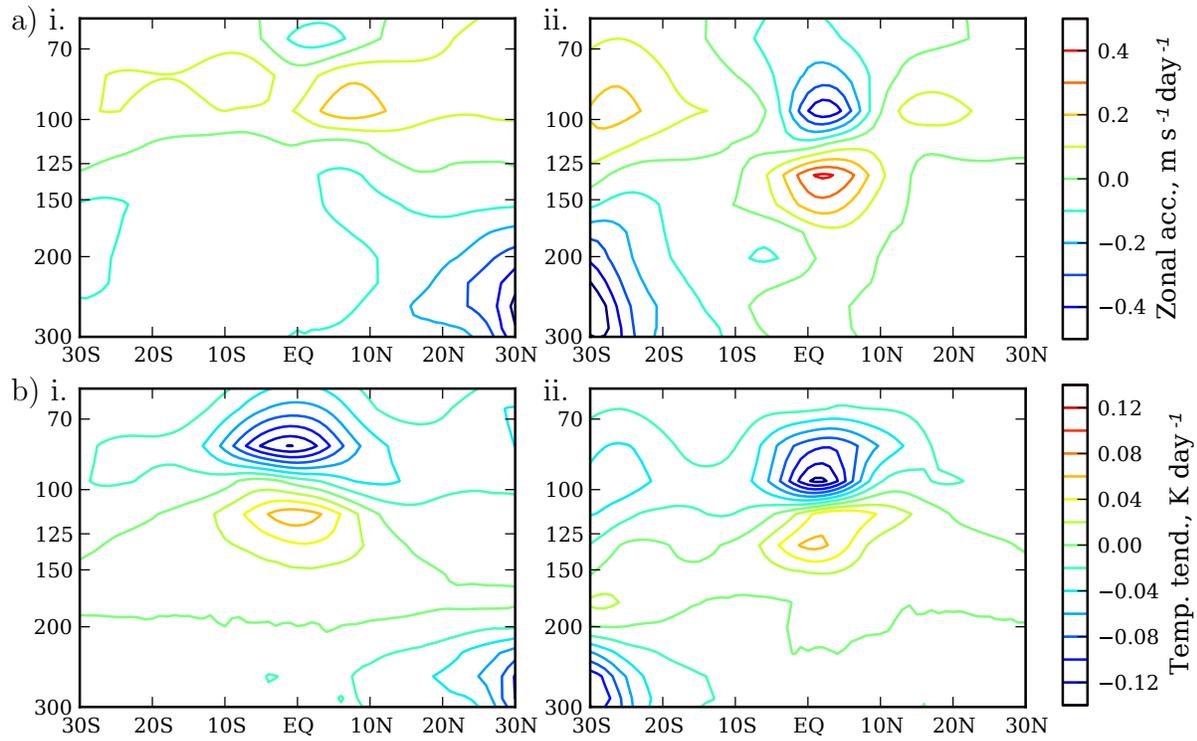


FIG. 2. Climatological zonal mean a) zonal acceleration and b) temperature tendency for i) DJF and ii) JJA computed using the rL scheme applied to ERA-Interim data from 1989 to 2009.

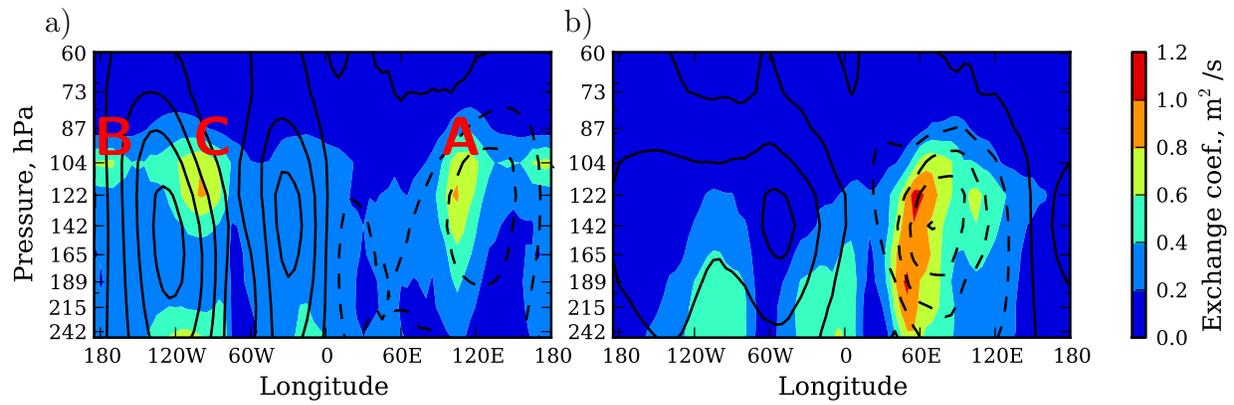


FIG. 3. Climatologically averaged exchange coefficient K_H according to the rL scheme for a) DJF and b) JJA averaged over 10°S – 10°N using ERA-Interim data from 1989 to 2009. Black contours show zonal wind. The labeled regions (“A”, “B”, “C”) of mixing in panel a) are referred to in the text.

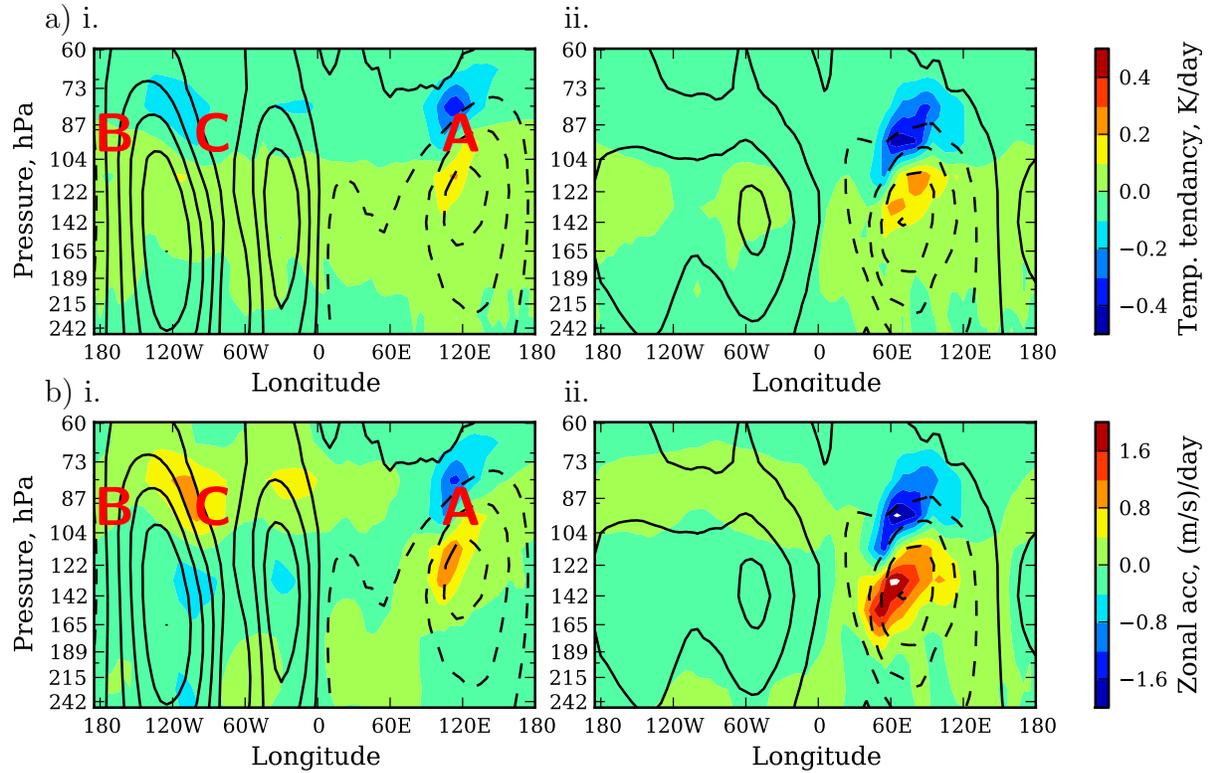


FIG. 4. a) Temperature tendency Q and b) zonal acceleration X due to the forcing terms arising from the revised Louis mixing scheme for i) DJF and ii) JJA averaged over 10°S – 10°N using ERA-Interim data from 1989 to 2009. Black contours show zonal wind. The regions A, B and C shown here are the same as those in Figure 3.

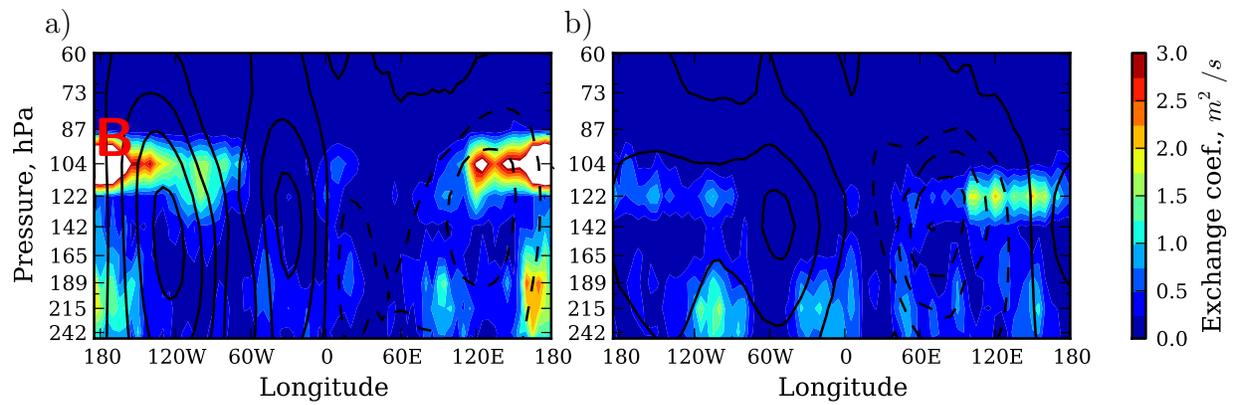


FIG. 5. As in Figure 3 but using the MO scheme. Region B is marked in the same location as in Figure 3a. Note that the color scale has been chosen to saturate before the maximum K_H in the DJF Pacific (approximately $10 m^2 s^{-1}$; regions above $3 m^2 s^{-1}$ are shown in white) to highlight the structure of K_H elsewhere in the domain.

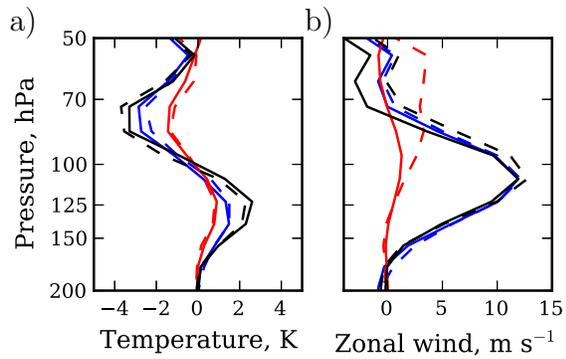


FIG. 6. Zonal average response to zonally symmetric forcings F_X^{sym} (blue solid), F_Q^{sym} (red solid) and both F_X^{sym} and F_Q^{sym} (black solid), in a) temperature and b) zonal wind, averaged over 10°N – 10°S , with HS94 background state. Similarly, the responses to the local forcings F_X (blue dashed), F_Q (red dashed) and both F_X and F_Q (black dashed).

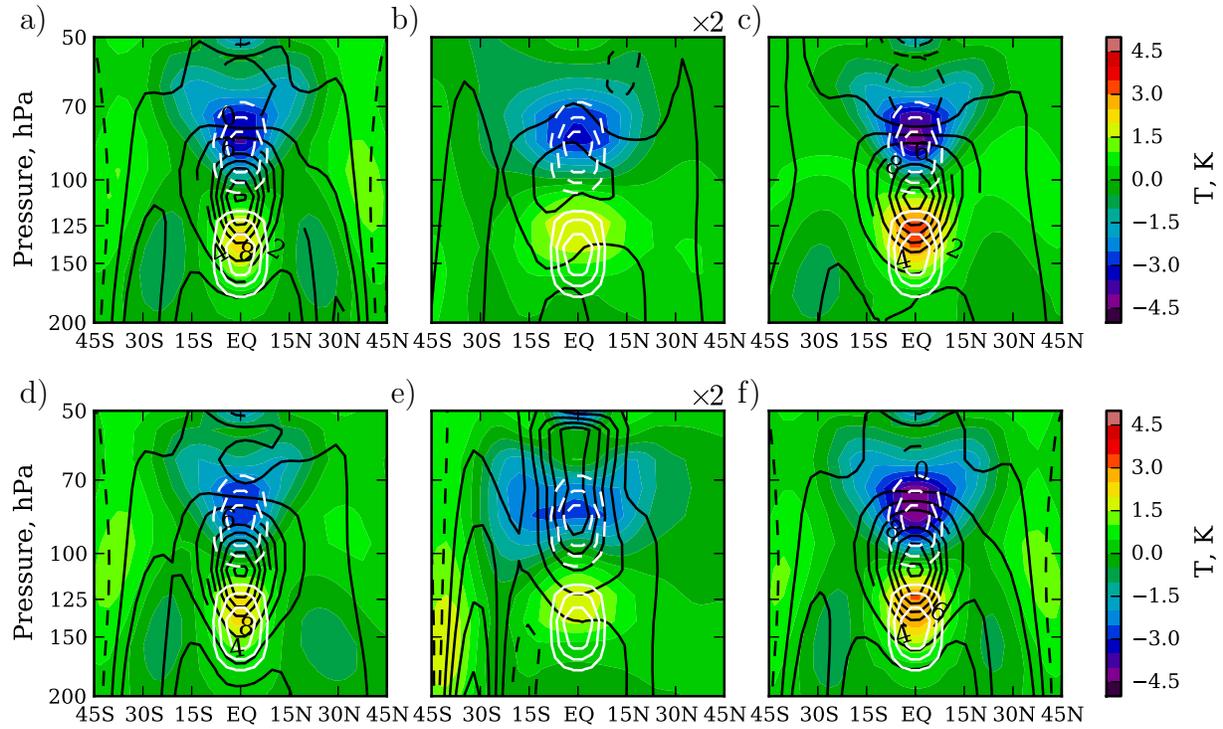


FIG. 7. Zonal mean temperature response $\delta\bar{T}$ (colors) and zonal mean wind response $\delta\bar{u}$ (black contours; contour spacing 2 m s^{-1} , 1 m s^{-1} in b and e) for forcings a) F_X^{sym} , b) F_Q^{sym} , c) both F_X^{sym} and F_Q^{sym} , d) F_X , e) F_Q , and f) both F_X and F_Q , with HS94 background state. The temperature color scale used in b) and e) is half that shown in the color bar. White contours show the structure of the forcing (normalized by amplitude.)

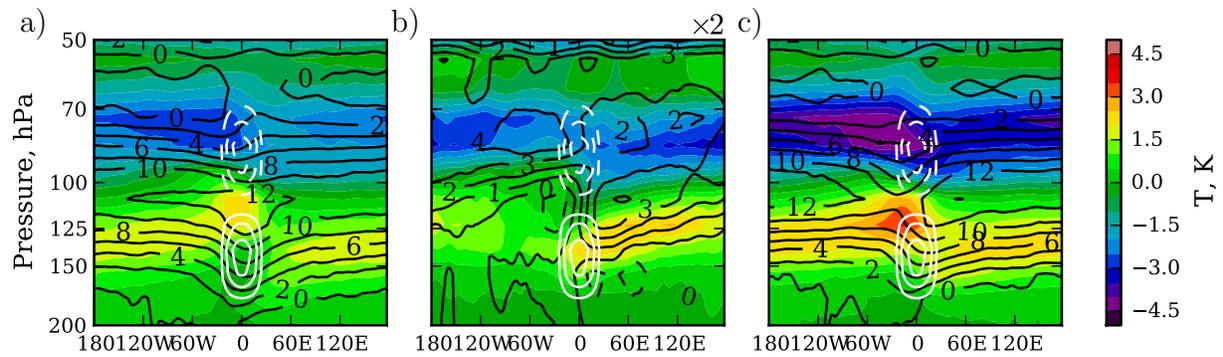


FIG. 8. Inner tropical (10°N – 10°S) average temperature response δT (colors) and zonal wind response δu (black contours; contour spacing 2 m s^{-1} , 1 m s^{-1} in b) for localized forcings a) F_X , b) F_Q , and c) both F_X and F_Q , with HS94 background state. The temperature color scale used in b) is half that shown in the color bar. White contours show the structure of F_X and F_Q .

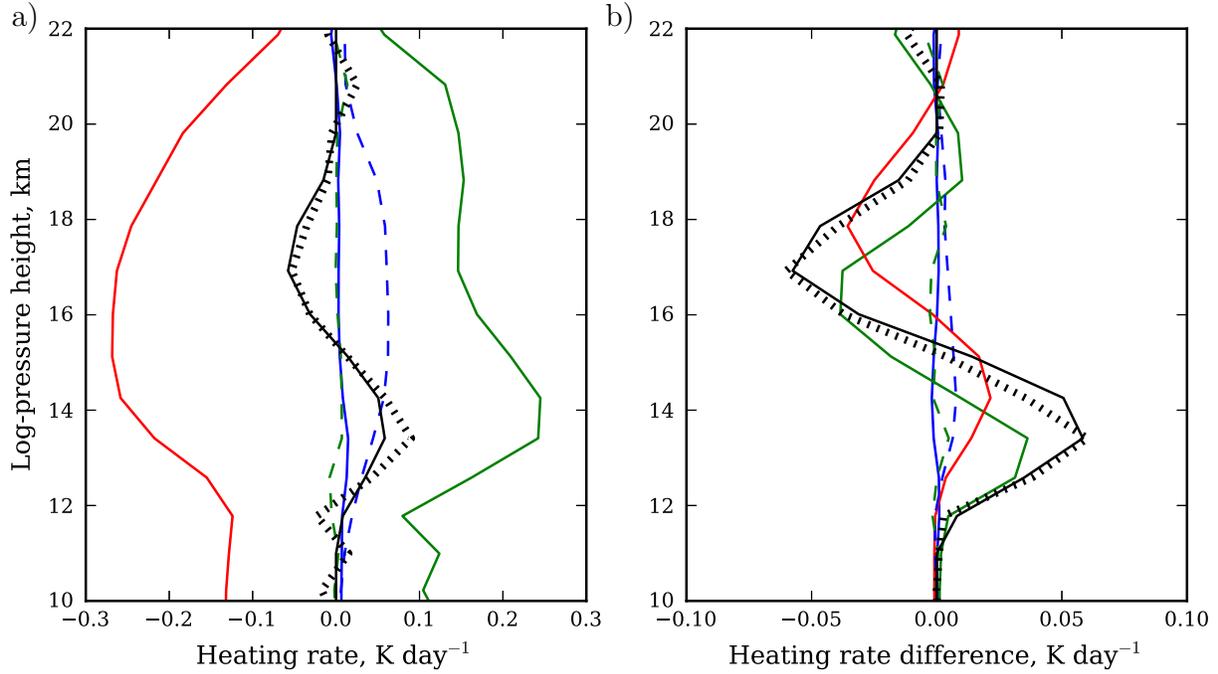


FIG. 9. a) Profiles of the terms in Eq. (5) (the time mean zonal mean buoyancy equation) averaged over $\pm 10^\circ$ for the run forced with F_Q^{sym} ; the meridional advection term (blue solid), the vertical advection term (green solid), the meridional eddy heat flux term (blue dashed), the vertical eddy heat flux term (green dashed), the Newtonian cooling term (red) and the imposed heating F_Q^{sym} (black solid) are shown. The sign of all terms except F_Q^{sym} are chosen to put them on the LHS of Eq. (5). The black dotted line shows the sum of all terms that balance the forcing term. b) Profiles of the difference between the forced and unforced runs for each quantity shown in a).

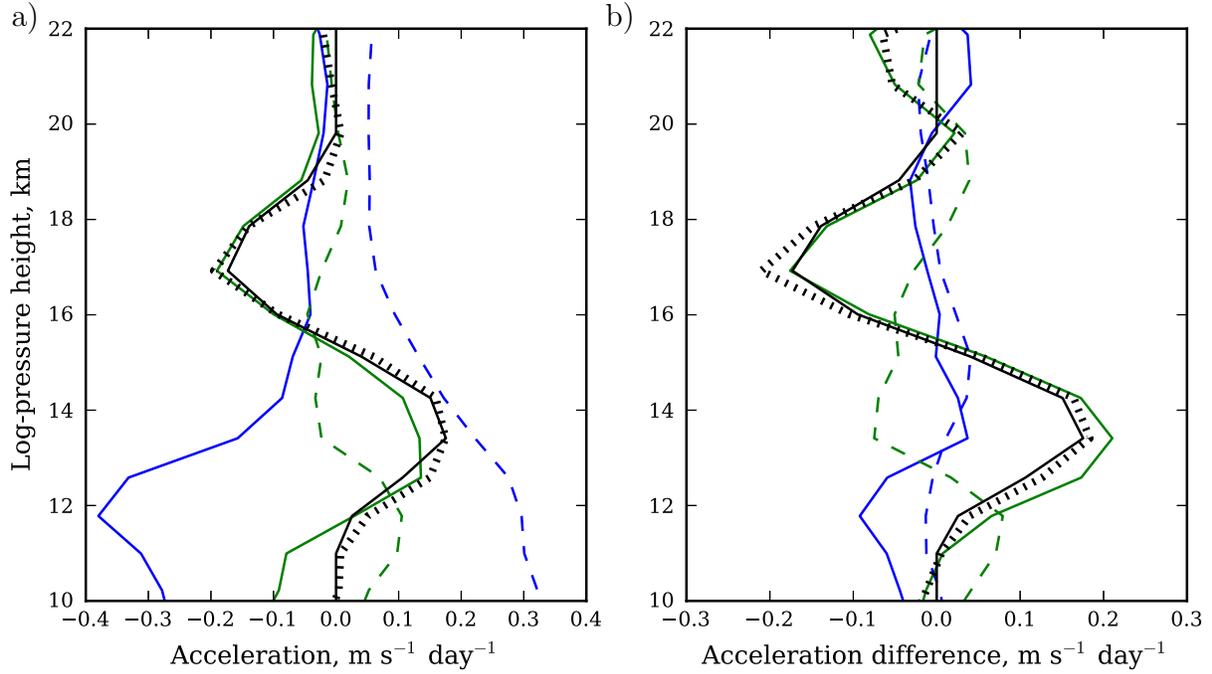


FIG. 10. a) Profiles of the terms in Eq. (9) (the time mean zonal mean zonal momentum equation) averaged over $\pm 10^\circ$ for the run forced with F_X^{sym} ; the sum of the meridional advection and Coriolis terms (blue solid), the vertical advection term (green solid), the meridional eddy momentum flux term (blue dashed), the vertical eddy momentum flux term (green dashed) and the imposed zonal acceleration F_X^{sym} (black solid) are shown. The black dotted line shows the sum of all terms that balance the forcing term. The sign of all terms except F_X^{sym} are chosen to put them on the LHS of Eq. (9). b) Profiles of the difference between the forced and unforced runs for each quantity shown in a).

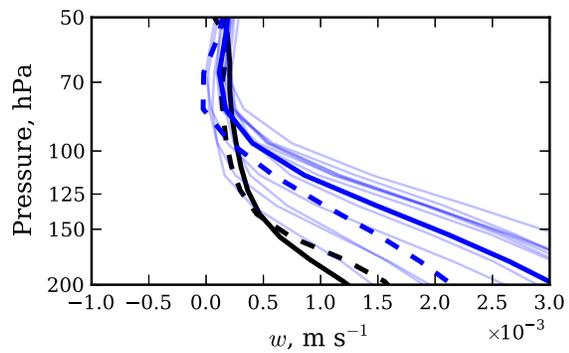


FIG. 11. Mean vertical velocity \bar{w} averaged over 10°N to 10°S (solid lines) and at the equator (dashed lines) for the HS94 run with no imposed forcing (black), and for ERA-Interim averaged over 1979 to 2012 (blue lines, thick line is average, thin lines are climatological annual cycle monthly averages.)

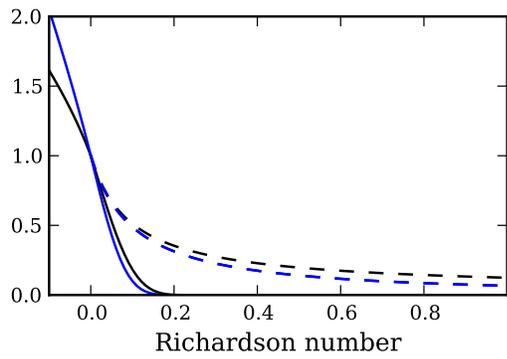


FIG. 12. $f_M(Ri)$ (black) and $f_H(Ri)$ (blue) in the MO scheme (solid) and rL scheme (dashed).

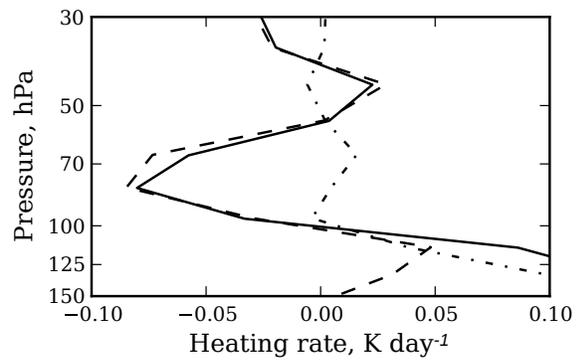


FIG. 13. Zonal mean ERA-Interim residual diabatic temperature tendency (solid), the temperature tendency due to vertical mixing as parametrised by the rL scheme (dashed), and the difference between these curves (dash-dot) averaged over January 2001 over 10°S–10°N.